Self-Tuning Control with Decoupling

A new method to design adaptive, self-tuning controllers is presented that incorporates decoupling for multivariable control problems. The algorithm is shown to result from two very different derivations. The first approach employs multiple single-input/single-output self-tuning controllers but with a classical decoupling scheme incorporated; the second approach utilizes a novel selection of design parameters in an existing multivariable self-tuning control method. The decoupling self-tuning controller is capable of handling unknown or time-varying time delays. Simulation studies utilizing two distillation column models show that the controller can provide excellent control even for highly nonlinear and highly interacting processes.

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Introduction

Multivariable chemical processes often exhibit characteristics that challenge traditional process control technology based on the use of multiple control loops, each loop incorporating a fixed-parameter three-mode controller. Problems arise whenever the process is:

1. Complex, which leads to difficulties in finding a suitable control structure (pairing of measured and manipulated variables) and in obtaining a useful dynamic characterization (model) from which to calculate suitable settings for the controller parameters
2. Nonlinear or time-varying, which leads to a changing dynamic model as the process and/or its operating conditions vary
3. Highly interacting, which usually produces unsatisfactory control due to interactions between the control loops

In recent years a greatly increasing research effort has been directed at the control of multiple-input/multiple-output processes to try to avoid these problems. Adaptive control methods have been singularly effective in dealing with some of the difficulties, particularly those involving poor or time-varying process models. Since their introduction in 1973, self-tuning adaptive controllers have been applied in a growing number of application areas. The basic idea behind self-tuners is to identify an empirical model from process input-output data obtained at each sampling instant, and to use the process model to adapt the controller parameters each time the model is updated. This approach appears to work very well with many different types of processes. The single-input/single-output self-tuning controllers developed originally by Åström and Wittenmark (1973) and Clarke and Gawthrop (1975) have been continually developed and refined; they also have been extended to a number of multivariable versions. Recent survey papers by Åström (1983) and Seborg et al. (1986) characterize the explosive growth in this field.

In many process control applications an important requirement is to be able to accommodate unknown, and time-varying, time delays in the process. Until quite recently, virtually all of the self-tuning techniques have required that time delays be known and constant; a paper by Prager and Wellstead (1980) appears to be the first to relax this restriction. Since then, several self-tuning techniques have incorporated the ability to deal with varying time delays effectively, including those of Vogel and Edgar (1982), Chien et al. (1984, 1985), and McDermott and Mellichamp (1984). The approach of Chien et al. moved a step further by also incorporating a time delay compensator that effectively increases the stability bounds of the controlled process.

The problem of interactions among control loops has been virtually ignored in the self-tuning literature, the exceptions being the multiloop decoupling, self-tuning controllers of McDermott and Mellichamp (1984) and Lang et al. (1986). However, the problem of decoupling in multivariable systems has been widely discussed in the chemical engineering literature, at least for the case of constant (nonadaptive) controllers. Two different approaches have been developed that attempt to *minimize* loop interactions through decoupling.

The first approach, known as internal decoupling, utilizes alternative pairings of manipulated and controlled variables to create new control loops that are less interactive. This method—discussed by Rijnsdorp (1965), Shinskey (1979), Wood and Berry (1973), Ryskamp (1980), and McAvoy (1983)—ordi-

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narily is preferred. However, an appropriate combination of variables may not be feasible if the existing control configuration cannot be altered or if the nature of the multivariable system precludes such a choice. The second approach utilizes knowledge about the dynamics of the multivariable system to synthesize a decoupling strategy that is incorporated in the control algorithm. This method has been called external decoupling (Jafarey and McAvoy, 1978). It might also be termed algorithmic decoupling. Such schemes have been widely studied by Luyben (1970), Waller (1974), Shinskey (1979), and Weischedel and McAvoy (1980).

Since the second approach requires an accurate dynamic model of the multivariable system in order to obtain satisfactory decoupling control, the use of adaptive control methods is very attractive. Any self-tuning method that generates an explicit multivariable process model at each sampling time through use of a suitable on-line parameter estimation algorithm can be used. Consequently, this paper describes the development of a decoupling strategy of the external type discussed above using single-variable self-tuning controllers with time delay compensation (STC-TDC) of Chien et al. (1984). The method is shown to be exactly equivalent to the multivariable STC-TDC of Chien et al. (1985) when a unique choice of controller weighting is made. Two simulated distillation systems are then used as examples to evaluate the new STC-TDC with decoupling. The first example is the transfer function model of Wood and Berry (1973) with provision for varying time delays and process gains. The second example is a physical model of a distillation column developed by Weischedel and McAvoy (1980). This model exhibits severe nonlinearities and a high degree of interaction between manipulated and controlled variables.

Decoupling Design Methods

Two basic approaches to external or algorithmic decoupling design are discussed in the distillation control literature, namely, ideal decoupling and simplified decoupling (Luyben, 1970; Waller, 1974; Shinskey, 1979; Weischedel and McAvoy, 1980). To illustrate these two approaches to external decoupling, we consider a system with two inputs and two outputs.

Ideal decoupling

The goal of ideal decoupling is to permit each control loop to behave as if the other loop were not present, i.e., the output response of each loop should be the same as would be obtained if the other loop were on manual control. Figure 1 shows a block diagram for ideal decoupling. (Luyben uses a different arrangement of this block diagram, but the result is equivalent to that in Figure 1.)

By specifying D_{12} and D_{21} as

$$D_{12} = -\frac{G_{p12}}{G_{p11}} \tag{1}$$

$$D_{21} = -\frac{G_{p21}}{G_{p22}} \tag{2}$$

the open-loop transfer function relating y_1 and u_{11} is

$$\frac{y_1}{u_{11}} = G_{p11} \tag{3}$$

A similar expression can be derived for the transfer function relating y_2 and u_{22}.

The control signals that affect the process can be calculated from the expression:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{G_{p12}}{G_{p11}} \\ \dfrac{G_{p21}}{G_{p22}} & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{11} \\ u_{22} \end{bmatrix} \tag{4}$$

Simplified decoupling

Waller (1974) suggests an alternative controller decoupling strategy to eliminate interactions between loops. Figure 2 shows the block diagram for his simplified decoupling.

The decoupling matrix D can be specified as one of the four choices given below:

$$D_1 = \begin{bmatrix} 1 & -\dfrac{G_{p12}}{G_{p11}} \\ -\dfrac{G_{p21}}{G_{p22}} & 1 \end{bmatrix} \tag{5}$$

$$D_2 = \begin{bmatrix} 1 & 1 \\ -\dfrac{G_{p21}}{G_{p22}} & -\dfrac{G_{p11}}{G_{p12}} \end{bmatrix} \tag{6}$$

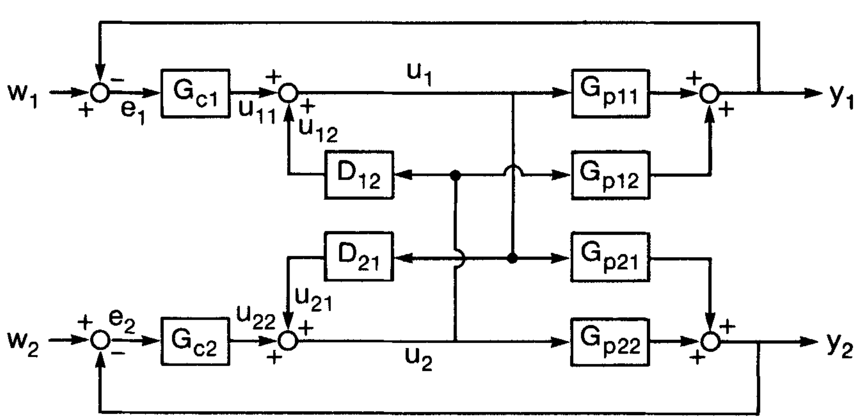

Figure 1. Ideal decoupling.

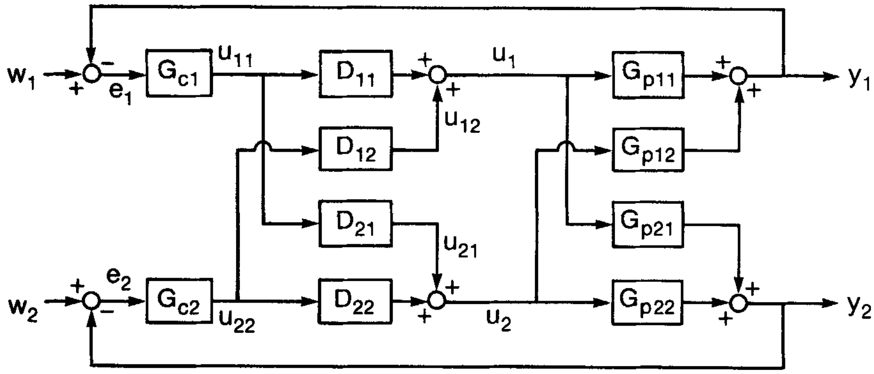

Figure 2. Simplified decoupling.

$$D_3 = \begin{bmatrix} -\dfrac{G_{p22}}{G_{p21}} & -\dfrac{G_{p12}}{G_{p11}} \\ 1 & 1 \end{bmatrix} \tag{7}$$

$$D_4 = \begin{bmatrix} -\dfrac{G_{p22}}{G_{p21}} & 1 \\ 1 & -\dfrac{G_{p11}}{G_{p12}} \end{bmatrix} \tag{8}$$

The open-loop transfer function relating y_1 and u_{11} when Eq. 5 is used for decoupling is:

$$\frac{y_1}{u_{11}} = G_{p_{11}}\left(1 - \frac{G_{p12}G_{p21}}{G_{p11}G_{p22}}\right) \tag{9}$$

Similar expressions can be derived for the transfer functions relating y_2 and y_{22} and for the alternative decoupler configurations in Eq. 6, 7, or 8.

The advantage of simplified decoupling is that one of the four decouplers is always physically realizable if the model orders are chosen to be equal. For example, if the process has off-diagonal time delays smaller than on-diagonal time delays, ideal decoupling will not be realizable, but one of the four choices of the simplified decoupling scheme always will be. The disadvantage of simplified decoupling is that it affects the apparent transfer function between each input-ouput pair; for example, compare Eqs. 3 and 9. This characteristic complicates the problem of designing G_{c1} and G_{c2}.

In the next section we propose two methods to eliminate interaction for the self-tuning controller with time delay compensation. The first method uses ideal decoupling with multiple single-input/single-output self-tuning controllers including time delay compensation (Chien et al., 1984). Ideal decoupling is used instead of simplified decoupling because the controller relations are much easier to obtain when using this decoupling scheme with the STC-TDC. The second method starts with the multivariable STC-TDC (Chien et al., 1985) and utilizes a particular choice of weighting matrix in the controller. The two methods unexpectedly reduce to identical closed-loop relationships.

Multiple Single-Variable STC-TDC's with Decoupler

We first illustrate this approach through the use of two single-input/single-output STC-TDC's with ideal decoupling. A detailed derivation of the single-variable version of the STC-TDC can be found in Chien et al. (1984). The process model to be considered is a multiple-input/multiple-output, discrete-time system with different time delays in individual loops described by

$$A(z^{-1})y(t) = B(z^{-1})u(t) + d + C(z^{-1})\xi(t) \tag{10}$$

where $y \in R^m$ is the output; $u \in R^m$ is the manipulated input; $d \in R^m$ is the constant, steady-state output response for a zero input signal; and $\xi \in R^m$ and $[\xi(t)]$ is a sequence of independent, normally distributed random vectors with zero means and with covariance matrix $[\xi(t)\xi(t)^T] = r_\xi$. Here the sampling instant is denoted by t ($t = 0, 1, 2, \ldots$).

A is an $m \times m$ diagonal polynomial matrix with elements:

$$A^{ii}(z^{-1}) = 1 + a_1^{ii}z^{-1} + \cdots + a_{n_{Aii}}^{ii}z^{-n_{Aii}} \tag{11}$$

B is an $m \times m$ polynomial matrix with elements:

$$B^{ij}(z^{-1}) = z^{-k^{ij}}(b_0^{ij} + b_1^{ij}z^{-1} + \cdots + b_{n_{Bij}}^{ij}z^{-n_{Bij}}),\quad b_0^{ij} \neq 0 \tag{12}$$

where k^{ij}, the time delay, is a positive integer expressed as a multiple of the sampling period ($k^{ij} \geq 1$) initially assumed to be known. C is an $m \times m$ diagonal polynomial matrix with elements

$$C^{ii}(z^{-1}) = 1 + c_1^{ii}z^{-1} + \cdots + c_{n_{Cii}}^{ii}z^{-n_{Cii}} \tag{13}$$

It is assumed that the roots of the determinant of $C(z^{-1})$ lie inside the unit circle in the complex z plane.

In terms of the process model shown in Figures 1 and 2:

$$G_{pij}(z^{-1}) = \frac{B^{ij}(z^{-1})}{A^{ii}(z^{-1})} \tag{14}$$

Using the single-loop STC-TDC strategy, we can obtain G_{c1} corresponding to G_{p11} and G_{c2} corresponding to G_{p22}. Briefly, the technique requires that a cost function be defined:

$$I_i = \mathscr{E}\{[P^{ii}(z^{-1})y_i(t + k_{ii}) - R^{ii}(z^{-1})w_i(t)]^2 + [Q^{ii}(z^{-1})u_i(t)]^2\} \tag{15}$$

where $w_i(t)$ is the set point for the ith loop and $P^{ii}(z^{-1})$, $Q^{ii}(z^{-1})$, and $R^{ii}(z^{-1})$ are weighting polynomials. A discrete version of the modified Smith predictor is substituted for $y_i(t + k_{ii})$

$$y_i(t + k_{ii}) = y_i(t) + \left(\frac{\Sigma_B^{ii} z^{-1}}{A^{ii}} - \frac{B^{ii} z^{-k}}{A^{ii}}\right) u_i(t) \tag{16}$$

Here Σ_B^{ii} is used to denote $\sum_{l=0}^{n_B^{ii}} b_l^{ii}$, i.e., a scalar element. Minimization of the cost function, subject to the physical realizability constraint imposed by the identity in z^{-1}

$$C^{ii}P^{ii} = A^{ii} + z^{-1}F^{ii} \tag{17}$$

leads to the control laws

$$(C^{11}P^{11}\Sigma_B^{11} + C^{11}A^{11}Q^{11} - F^{11}B^{11})u_{11} = A^{11}(C^{11}R^{11}w_1 - F^{11}y_1) \tag{18}$$

$$(C^{22}P^{22}\Sigma_B^{22} + C^{22}A^{22}Q^{22} - F^{22}B^{22})u_{22} = A^{22}(C^{22}R^{22}w_2 - F^{22}y_2) \tag{19}$$

Note that the use of the modified Smith predictor changes the identity in Eq. 17 from its usual form involving $z^{-k_{ii}}F$ to the present form. The advantages of this approach, particularly its enhanced stability characteristics, have been emphasized earlier (Chien et al., 1984).

Application of the ideal decoupling expressions given by Eq. 4 leads to the decoupling relations; for the 2 × 2 example, these are

$$(B^{11}B^{22} - B^{12}B^{21})u_1 = B^{11}B^{22}u_{11} - B^{12}B^{22}u_{22} \tag{20}$$

$$(B^{11}B^{22} - B^{12}B^{21})u_2 = -B^{11}B^{21}u_{11} + B^{11}B^{22}u_{22} \tag{21}$$

Note that these equations are applicable when the off-diagonal delays are larger than the on-diagonal delays, i.e., $k^{11} < k^{12}$ and $k^{22} < k^{21}$. The closed-loop equations can be derived by combining Eqs. 18 through 21 with the process model to obtain the relationships between outputs and set points:

$$y_1 = \frac{R^{11}B^{11}}{P^{11}\Sigma_B^{11} + A^{11}Q^{11}} w_1 \tag{22}$$

$$y_2 = \frac{R^{22}B^{22}}{P^{22}\Sigma_B^{22} + A^{22}Q^{22}} w_2 \tag{23}$$

Figure 3 shows the block diagram for the 2 × 2 example. Here, two single-input/single-output (SISO) STC-TDC's are used with the ideal decoupling relations. From the standpoint of implementation, the process model can be identified at each sampling instant by any suitable technique, for example, a multivariable recursive least-squares method utilizing the known input-output data, to develop explicit model parameters. In this work, extended least squares with UD factorization has been used. Once the model parameters are available, each controller equation (e.g., Eq. 18) can be developed independently of the other(s). Development of the decoupling relationships (e.g., Eq. 20) can follow. Finally, the inputs to the process—$u_1, u_2, \ldots$—can be computed and applied. Implicit in this development, as in all STC developments, is that the estimated parameters can be substituted for the actual parameters in Eqs. 16 through 21. The most restrictive assumption in this development, the availability of known and unvarying time delays, will be dealt with below.

Multivariable STC-TDC

An alternate decoupling approach can be based on a novel selection of the Q matrix in the multivariable STC-TDC controller design. A detailed derivation of this multivariable controller without decoupling can be found in Chien et al. (1985). The resulting control law is

$$u(t) = (CP\Sigma_B + CAQ - FB)^{-1}A\epsilon \tag{24}$$

where

$$\epsilon = CRw - Fy - d \tag{25}$$

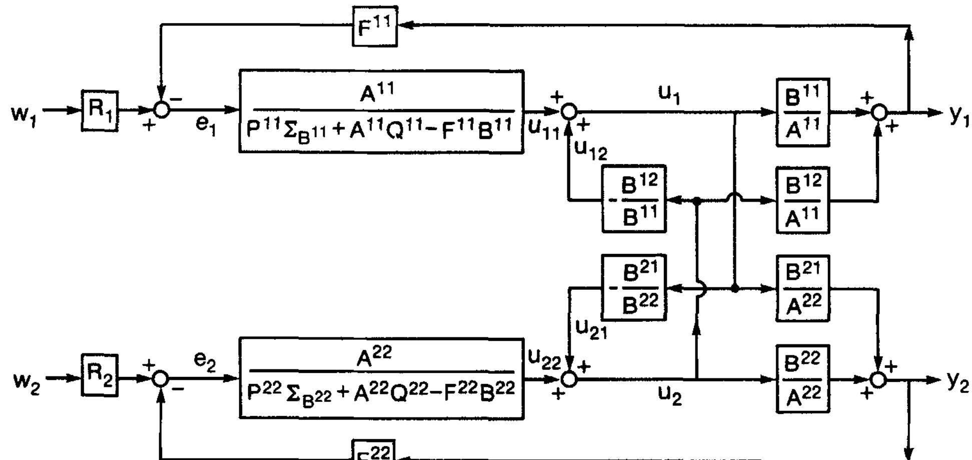

Figure 3. Two SISO STC-TDC's with ideal decoupling.

In Eqs. 24 and 25 u, ϵ, w, y, and d are vectors. The remaining elements are matrices analogous to the SISO quantities. Note that Q, the input weighting matrix, in Eq. 24 is full rather than diagonal as in the previous development.

The closed-loop equation is obtained by substituting Eq. 24 into the process model, Eq. 10, and simplifying to obtain

$$y(t) = [(P\Sigma_B + AQ)B^{-1}A]^{-1}\{ARw(t) + A(1)Q(1)B^{-1}(1)d + [(P\Sigma_B + AQ)B^{-1}C - F]\xi(t)\} \quad (26)$$

The Appendix gives the details of this derivation. The closed-loop relationship between outputs and set points is then

$$y(t) = A^{-1}B(P\Sigma_B + AQ)^{-1}ARw(t) \quad (27)$$

By inspection of Eq. 27, we see there will be no interaction if the matrix product $B(P\Sigma_B + AQ)^{-1}$ is diagonal. Since Q is a user-specified weighting matrix, let the element Q^{ij} be a rational function:

$$Q^{ij}(z^{-1}) = \frac{Q_n^{ij}(z^{-1})}{Q_d^{ij}(z^{-1})} \quad (28)$$

Again taking a two-input/two-output system as an example, the condition for $B(P\Sigma_B + AQ)^{-1}$ to be diagonal is:

$$B^{11}A^{11}Q_n^{12} = (B^{12}P^{11}\Sigma_B^{11} + B^{12}A^{11}Q^{11} - B^{11}P^{11}\Sigma_B^{12})Q_d^{12} \quad (29)$$

$$B^{22}A^{22}Q_n^{21} = (B^{21}P^{22}\Sigma_B^{22} + B^{21}A^{22}Q^{22} - B^{22}P^{22}\Sigma_B^{21})Q_d^{21} \quad (30)$$

Solving Eq. 29 for Q_n^{12} and Q_d^{12}, we obtain

$$Q_n^{12} = z^{-(k^{12}-k^{11})}(\bar{B}^{12}P^{11}\Sigma_B^{11} + \bar{B}^{12}A^{11}Q^{11}) - \bar{B}^{11}P^{11}\Sigma_B^{12} \quad (31)$$

$$Q_d^{12} = \bar{B}^{11}A^{11} \quad (32)$$

where

$$\bar{B}^{ij} = b_0^{ij} + b_1^{ij}z^{-1} + \cdots + b_{n_{Bij}}^{ij}z^{-n_B^{ij}} \quad (33)$$

Similar expressions can be obtained for Q_n^{21} and Q_d^{21}.

Substituting the control law, Eq. 24, and the expressions for Q_n^{12}, Q_d^{12}, Q_n^{21}, and Q_d^{21} into the process model, Eq. 10, the relationship between the outputs and set points is identical to Eqs. 22 and 23. Thus these two methods generate identical control laws and closed-loop relations even though they are based on different approaches.

Unknown and Varying Time Delays

The above analysis is based on a process model with known time delays. If the actual time delays are not known but the range of each time delay is known, $k_{min}^{ij} \le k^{ij} \le k_{max}^{ij}$, then the B polynomial matrix in the process model, Eq. 10, can be expanded to accommodate the extra delays in excess of k_{min}^{ij} as follows:

$$B_E^{ij}(z^{-1}) = z^{-k_{min}^{ij}}(\tilde{b}_0^{ij} + \tilde{b}_1^{ij}z^{-1} + \cdots + \tilde{b}_{r^{ij}}^{ij}z^{-r^{ij}}) \quad (34)$$

with r^{ij} determined by

$$r^{ij} = k_{max}^{ij} - k_{min}^{ij} + n_B^{ij} \quad (35)$$

Using the expanded polynomial matrix B_E, the decoupling equations, Eqs. 20 and 21, become:

$$(B_E^{11}B_E^{22} - B_E^{12}B_E^{21})u_1 = B_E^{11}B_E^{22}u_{11} - B_E^{12}B_E^{22}u_{22} \quad (36)$$

$$(B_E^{11}B_E^{22} - B_E^{12}B_E^{21})u_2 = - B_E^{11}B_E^{21}u_{11} + B_E^{11}B_E^{22}u_{22} \quad (37)$$

If the actual time delays, k^{11} and k^{22}, are greater than the minimum expected time delays, k_{min}^{11} and k_{min}^{22}, the estimated leading coefficients of B_E^{11} and B_E^{22} will be close to zero. Equations 36 and 37 will generate extremely large (unstable) control signals. In order to obtain stable control, a conservative criterion should be used to eliminate leading coefficients of B_E^{11} and B_E^{22} when the values are close to zero. A suitable empirical criterion was found to be:

$$\frac{|\tilde{b}_j^{ii}|}{\max|\tilde{b}_j^{ii}|} > 0.2 \quad j = 0, \ldots, r^{ii}; \quad i = 1, 2 \quad (38)$$

If the lead coefficient fails to satisfy the criterion, this coefficient should be dropped and the corresponding time delay, k_{min}^{ij}, in that polynomial increased by one. The procedure is repeated with successive coefficients in the polynomial until the criterion is satisfied. The remaining coefficients of the polynomial then should be scaled so as to retain the process gain of the original estimated process model.

Simulation Results

In this section, simulation results for two distillation column models are presented. Comparisons of the STC-TDC with decoupling are made with several previously reported controllers.

Example 1

The performance of the STC-TDC with decoupling is first demonstrated using the two-input/two-output model developed by Wood and Berry (1973) for a pilot-scale distillation column that separates a mixture of methanol and water. This model has been used in a number of other multivariable control studies, including those by Shah and Fisher (1978), Ogunnaike and Ray (1979), Vogel and Edgar (1982), and McDermott and Mellichamp (1984). For the typical steady state operating conditions in Table 1, Wood and Berry (1973) reported the following empirical transfer function model

$$y(s) = G_p(s)u(s) + G_L(s)L(s) \quad (39)$$

Table 1. Typical Steady State Operating Conditions for Example 1

Product split	0.5–96.0 wt. % methanol
No. of trays	8
Reflux flow rate	0.885 kg/min
Steam flow rate	0.776 kg/min
Feed flow rate	1.11 kg/min
Feed composition	46.5 wt. % methanol

where the inputs and outputs are deviation variables defined as

y_1 = overhead product composition (wt. % methanol)

y_2 = bottoms product composition (wt. % methanol)

u_1 = reflux flow rate (kg/min)

u_2 = steam flow rate (kg/min)

L = feed flow rate (kg/min)

The transfer function matrices, $G_p(s)$ and $G_L(s)$, are given by

$$G_p(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s + 1} & \frac{-18.9e^{-3s}}{21.0s + 1} \\ \frac{6.6e^{-7s}}{10.9s + 1} & \frac{-19.4e^{-3s}}{14.4s + 1} \end{bmatrix} \quad (40)$$

$$G_L(s) = \begin{bmatrix} \frac{3.8e^{-8.1s}}{14.9s + 1} \\ \frac{4.9e^{-3.4s}}{13.2s + 1} \end{bmatrix} \quad (41)$$

This process was simulated after first converting the transfer function model into a state-space model as discussed by Ogunnaike and Ray (1979). For control purposes, a sampling period of 1 min was chosen; process noise was simulated by adding a white noise signal with a standard deviation of 0.025 to the feed flow rate input, L.

A particular test sequence was employed to examine the performance of three control strategies. A step change of 0.5% was made in the set point of the overhead composition at $t = 20$ min. After returning to the original set point at $t = 70$ min, a step change of 0.154 kg/min in the feed flow rate (approx. 14%) was made at $t = 120$ min. Simultaneously with the feed flow rate increase, the process gain and time delays were decreased so that the process model in Eq. 40 was replaced by

$$G_p^*(s) = \begin{bmatrix} \frac{6.1}{16.7s + 1} & \frac{-8.4e^{-2s}}{21.0s + 1} \\ \frac{3.5e^{-4s}}{10.9s + 1} & \frac{-8.8e^{-s}}{14.4s + 1} \end{bmatrix} \quad (42)$$

Finally, the sequence of set point changes in the overhead composition was repeated with an increase at $t = 200$ min, followed by a decrease at $t = 250$ min.

This test sequence was first introduced by Vogel and Edgar (1982) and is used here to obtain results that can be compared. The assumption of instantaneous gain and time delay changes is very approximate with respect to dynamic operations of the hypothetical column after an increase in feed flow rate; it does, however, furnish a drastic test case for the adaptive controller because of the suddenness of the model change.

The standard self-tuning controller with *no provision* for time delay changes (STC) was considered first, using the test procedure described above. The STC design, based on the original set of time delays, was found to go unstable as soon as the time delays changed. (No plots are shown.) Figure 4 shows the results of the standard self-tuning controller *with provision* for time delay changes, i.e., with expanded B polynomial matrix, using assumed minimum time delays of $k^{11}_{min} = 1$, $k^{12}_{min} = 3$, $k^{21}_{min} = 5$, and $k^{22}_{min} = 2$. As discussed by Chien et al. (1984), the stable region of tuning parameter Q is quite small; in this case λ (the single parameter in Q, ordinarily picked by a simple trial-and-error simulation procedure) has to be chosen very large in order to satisfy stability criteria. From Figure 4 we note that although the first series of set point changes is stable, the response is oscillatory. In the second series of changes, the system response is very sluggish due to the required use of large λ in the Q polynomial matrix. Note that standard proportional-integral controllers, even with decoupling, will not work for this test case because the large changes in gains and time delays completely detune the controllers and the decouplers, all of which must be conservatively designed on the basis of a fixed model.

Figure 5 shows the results of applying the multiple-input/

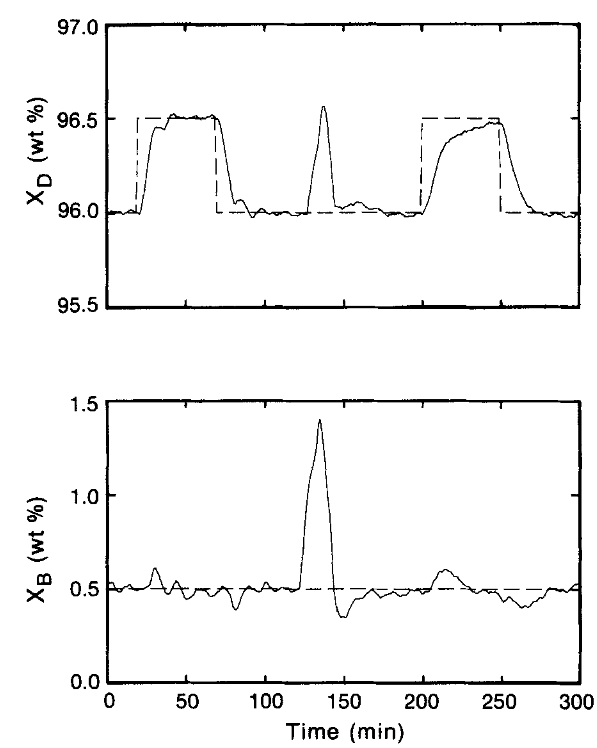

Figure 4. Standard STC with provision for time delay changes.
$P = (1 - 0.8z^{-1})I$; $Q = 3(1 - z^{-1})\,\mathrm{diag}(1, -1)I$; $R = 0.2I$

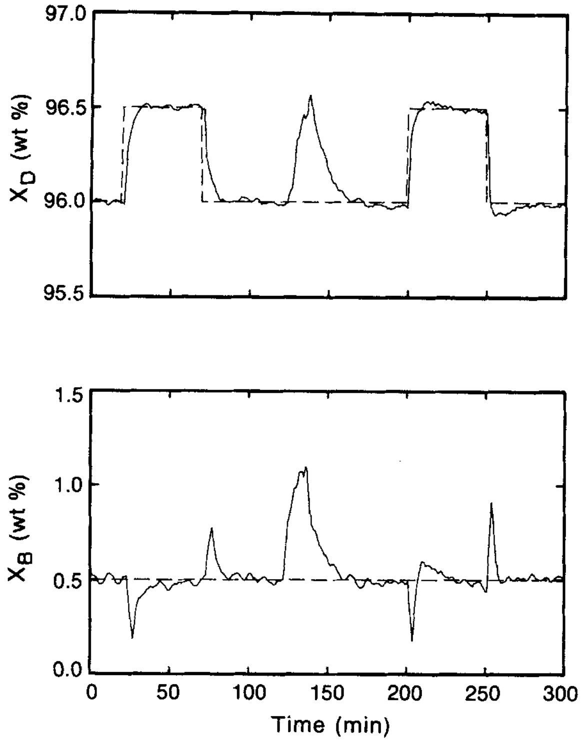

Figure 5. MIMO STC with time delay compensation.
$P = (1 - 0.8z^{-1})I$; $Q = 0.01(1 - z^{-1})\,\mathrm{diag}(1, -1)I$; $R = 0.2I$

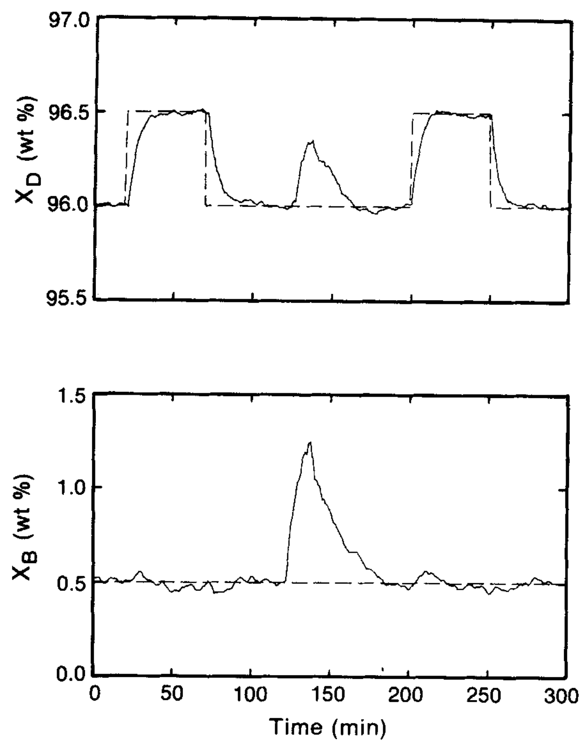

Figure 6. STC with time delay compensation, with decoupling.
$P = (1 - 0.8z^{-1})I$; $Q = 0.01(1 - z^{-1})\,\mathrm{diag}(1, -1)I$; $R = 0.2I$

multiple-output (MIMO) STC-TDC of Chien et al. (1985). The closed-loop performance is quite good, indicating that this controller can handle both time delay and gain changes. However, significant loop interactions develop when the set points are changed. In contrast, Figure 6 shows the application of the STC-TDC with decoupling for the same test sequence. Note that this adaptive system is almost completely decoupled and that both its set point tracking and load response characteristics are excellent.

For this example and the following ones, the STC weighting matrices—P, Q, R—were chosen following the guidelines given in Sec. 4.7 of Chien (1985).

Example 2

The performance of the STC-TDC with decoupling is further demonstrated and compared with alternative controllers using a distillation column dynamic model developed by Weischedel and McAvoy (1980). This model describes a hypothetical methanol-ethanol column whose characteristics are summarized in Table 2; the model was derived by writing overall and component mass balances for each tray in the column. Thus the model consists of $2N + 2$ (56) nonlinear differential equations, where N is the number of trays, plus a set of algebraic energy equations, relations for vapor-liquid equilibria, physical property data, and related factors.

The column, usually referred to as column C, is highly nonlinear and exhibits a large degree of interaction. For example, at nominal operating conditions the relative gain between top composition and reflux flow rate is $\lambda_{11} = 18.5$. More detailed descriptions of the column model and the computer simulation are available elsewhere (Weischedel, 1980; Weischedel and McAvoy, 1980).

In our simulation studies using this nonlinear dynamic model, the overhead and bottoms compositions were controlled by manipulating the reflux flow rate and the vapor flow rate from the reboiler. Analyzer dynamics were modeled as a pure time delay of 4 min. Process noise was introduced by adding a white noise signal, with zero mean and a standard deviation of 0.01, to the feed flow rate.

The following set point test sequence was employed to examine the performance of three alternative control strategies. A step change of 0.5% was first made in the set point of the bottoms composition controller. The set point was then returned to its nominal value. After a second similar step change was made in the bottoms composition, the set point of the overhead composition controller was changed to 0.995 in order to demonstrate the controller performance under different operating conditions.

The performance of the conventional decoupling control scheme (without self-tuning) developed by Weischedel and McAvoy (1980) was first investigated. The transfer function matrix model reported by Weischedel and McAvoy, with a time delay of 4 min incorporated to represent analyzer dynamics, was

Table 2. Column C Information

Mixture	Methanol-ethanol
Product split	0.01–0.99 mol frac. methanol
No. of trays	27
Reflux flow rate	3.384 mol/min
Vapor flow from reboiler	3.856 mol/min
Feed flow rate	1.50 mol/min
Feed composition	0.5 mol frac. methanol

used to design the simplified decouplers in Eq. 5 and to tune two conventional PI controllers. Using the effective open-loop transfer functions for the process and decouplers, as shown in Eq. 9, Ziegler-Nichols settings for the top loop were found to be $K_c = 178.8$ mol/min, $\tau_I = 16.8$ min; for the bottom loop, $K_c = -253.6$ mol/min, $\tau_I = 17.9$ min. These large controller gain values arise from the extra factor in Eq. 9, which results in a very small multiplier for this case.

The above PI controller settings, with simplified decoupling, were tested first; the resulting closed-loop system was unstable. By trial and error, the top and bottom loop controller gains were reduced to 1/4 and 1/6 of the original settings, respectively, to obtain the reasonable closed-loop responses of Figure 7. The response of the bottoms set-point change is quite good, but interaction effects still are evident in the overhead composition. After the last set point change in the overhead composition, the response began to oscillate. Due to the slight shift in operating conditions, the linear model reported by Weischedel and McAvoy (1980) cannot adequately describe the highly nonlinear column.

The standard STC was investigated as the next candidate controller. A detailed description of the method can be found in Chien et al. (1983). The sampling period was chosen to be 4 min and the predictive model was based on the following model parameters: $n_a = 2$, $n_b = 1$, $n_c = 1$, $k = 2$. As indicated in Figure 8, the STC adapts to the operating condition changes quite well except that loop interactions are present during set point changes.

Finally, Figure 9 shows the performance of the STC-TDC with decoupling for this test sequence. The sampling period was again chosen to be 4 min, and the process model utilized $n_A^{ii} = 2$, $r^{ii} = 2$, $n_C^{ii} = 0$, and $k_{min}^{ii} = 1$ in Eqs. 10 and 34. The responses shown in Figure 9 are excellent, with the system almost completely decoupled. This example shows that the STC-TDC with decoupling works well even for a highly nonlinear, interacting column.

This example also can indicate the need to have good *a priori* estimates of the time delays for both the PI with decoupling con-

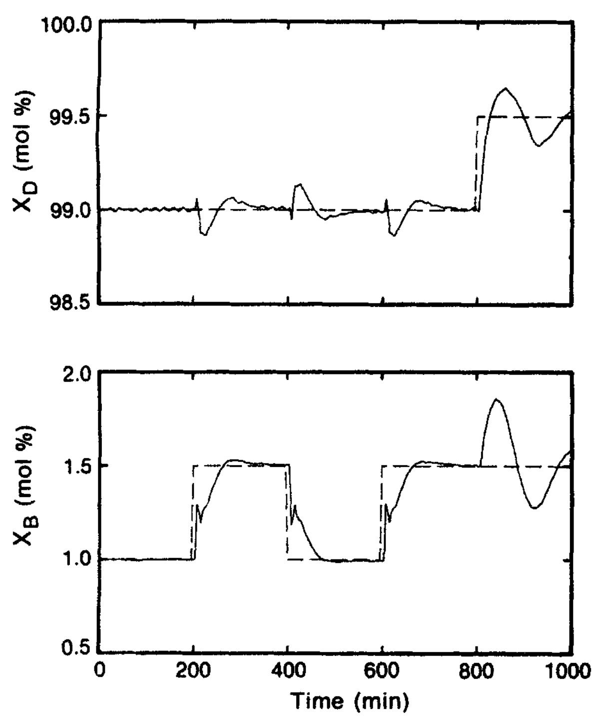

Figure 7. Dual-loop PI controller with simplified decoupling.

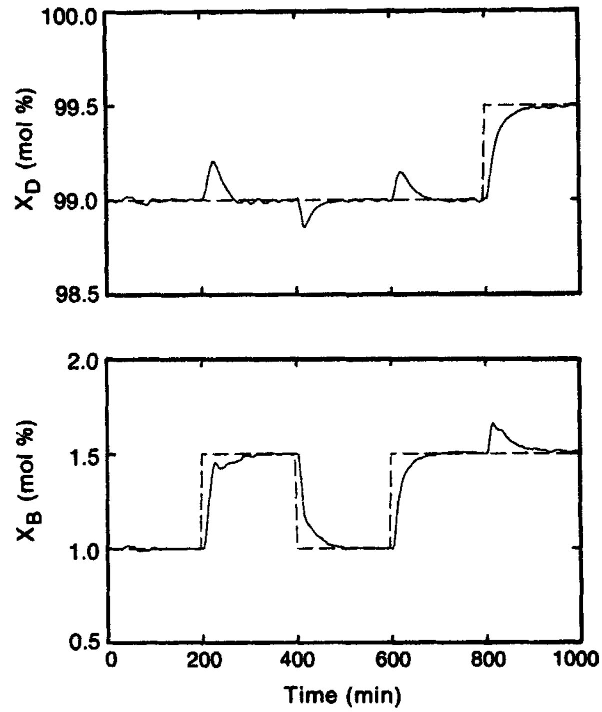

Figure 8. Self-tuning controller.
$P = (1 - 0.7z^{-1})I$; $Q = 0.02(1 - z^{-1})\,\mathrm{diag}(1, -1)$; $R = 0.2I$

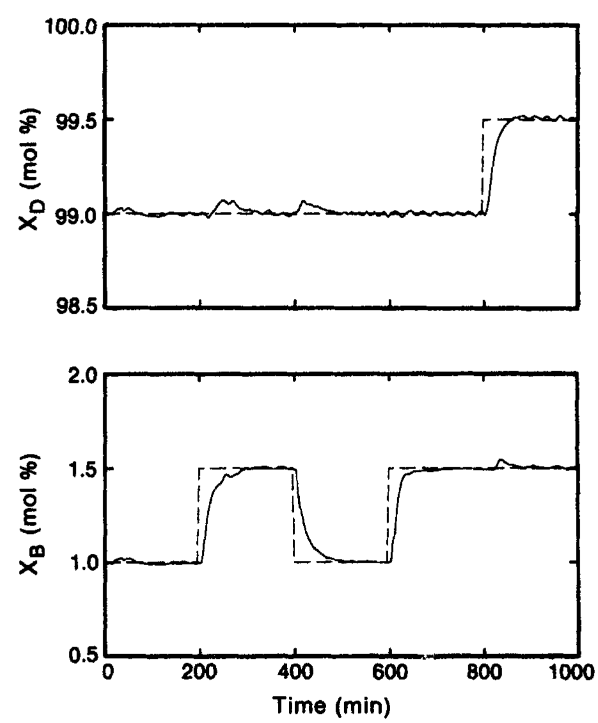

Figure 9. STC with time delay compensation, with decoupling.
$P = (1 - 0.7z^{-1})I$; $Q = 0.02(1 - z^{-1})\,\mathrm{diag}(1, -1)I$; $R = 0.3I$

trol system and for the standard STC. In the former case, the two primary (PI) controllers and the decouplers are designed (tuned) carefully for particular values of the time delays. In the latter case, the model structure must be matched carefully to the assumed values of the time delays. In either instance a change in the actual delays from design values—for example, the analyzer time becomes different from the assumed value of 4 min—will result in a deterioration in the system responses, although the standard STC will exhibit less deterioration than the PI control system. In contrast, the STC-TDC is designed to handle such discrepancies, estimating different values of the elements in the B polynomials as the process time delay(s) change and modifying the controller response accordingly. Since in this case the estimated process delay model can match the process delays exactly, the controller can deal with time delay changes directly and efficiently.

Conclusions

Decoupling methods have been proposed to eliminate interactions between control loops in multivariable processes that are controlled by either of the two self-tuning controllers incorporating time delay compensation (STC-TDC) of Chien et al. (1984, 1985). It is concluded that the two design methods, although based on very different starting points, lead to the same control law. Simulation studies of two distillation columns show:

1. The standard multivariable STC does not work well if unknown time delay variations are encountered
2. The PI control system (with simplified decoupling) of Weischedel and McAvoy (1980) exhibits oscillatory responses for a high-purity distillation column when the operating conditions are changed
3. The STC-TDC with decoupling performs well with both these distillation column models and the respective closed-loop systems are seen to be almost completely decoupled.

These two examples clearly demonstrate the superior performance of this new controller over traditional multiloop PI controllers, even with decoupling, and over adaptive multivariable controllers that do not incorporate time delay compensation and/or decoupling.

Notation

A, B, C = process z-transform polynomial matrices
A^{ii}, B^{ij}, C^{ii} = elements of the process polynomial matrices
a_l^{ii} = coefficient of A^{ii} polynomial
$\overline{B}^{ij}$ = process polynomial of B^{ij} without time delay term
$\overline{B}_E^{ij}$ = expanded polynomial of B^{ij}
b_l^{ij} = coefficient of B^{ij} polynomial
$\overline{b}_l^{ij}$ = coefficient of B_E^{ij} polynomial
c_l^{ii} = coefficient of C^{ii} polynomial
D = decoupling matrix
D_{ij} = (i, j) element of D
d = bias term in process model
e_i = error signal of loop i
$\mathcal{E}$ = expectation operator
F = polynomial matrix
F^{ii} = (i, i) element of F
G_{ci} = controller transfer function for loop i
G_L = disturbance transfer function matrix
G_p = process transfer function matrix
G_{pij} = (i, j) element of process transfer function matrix G_p
$G_p^*(s)$ = process transfer function matrix (after load change)
K_c = proportional gain of PI controller
k^{ij} = time delay between output i and input j ($k^{ij} \geq 1$)
k_{max}^{ij} = maximum expected time delay between output i and input j
k_{min}^{ij} = minimum expected time delay between output i and input j
L = load disturbance
m = dimension of multivariable system
N = number of trays
n_Aii = order of A^{ii} polynomial
n_a = order of A polynomial matrix
n_Bij = order of B^{ij} polynomial
n_b = order of B polynomial matrix
n_Cii = order of C^{ii} polynomial
n_c = order of C polynomial matrix
P = polynomial matrix
p^{ii} = polynomial
Q = polynomial matrix
Q^{ij} = polynomial or rational function
Q_d^{ij} = denominator polynomial of Q^{ij}
Q_n^{ij} = numerator polynomial of Q^{ij}
R = polynomial matrix
R^{ii} = polynomial
r^{ij} = order of B_E^{ij} polynomial
r_ξ = covariance matrix for white noise
s = Laplace transform variable
t = sampling instant
u = manipulated input vector
u_i = manipulated input for loop i
u_{ii} = controller output for loop i
u_{ij} = output from the decoupler D_{ij}
w_i = setpoint for loop i
y = measured output vector
y_i = measured output for loop i
z = z-transform variable

Greek letters

ϵ = error vector
ξ = white noise vector
λ = scalar parameter in Q^{ij}
Σ_B^{ii} = (i, i) scalar element $= \sum_{l=0}^{n_B ii} B_l^{ii}$
Σ_B = matrix of scalar elements
λ_{ii} = relative gain array element
τ_I = reset time of PI controller

Appendix A: Derivation of Closed-loop Eq. 26

Substituting Eqs. 24 and 25 into Eq. 1 yields

$$Ay(t) = B(CP\Sigma_B + CAQ - FB)^{-1}A \cdot [CRw(t) - Fy(t) - d] + d + C\xi(t) \quad \text{(A1)}$$

Collecting $y(t)$ terms on the lefthand side gives

$$[A + B(CP\Sigma_B + CAQ - FB)^{-1}AF]y(t) = B(CP\Sigma_B + CAQ - FB)^{-1}A \cdot [CRw(t) - d] + d + C\xi(t) \quad \text{(A2)}$$

Premultiplying by $(CP\Sigma_B + CAQ - FB)B^{-1}$ gives

$$[(CP\Sigma_B + CAQ - FB)B^{-1}A + AF]y(t) = ACRw(t) + [(CP\Sigma_B + CAQ - FB)B^{-1} - A]d + (CP\Sigma_B + CAQ - FB)B^{-1}C\xi(t) \quad \text{(A3)}$$

Since A, C, and F are all diagonal matrices, Eq. A3 becomes

$$[(CP\Sigma_B + CAQ)B^{-1}Ay(t) = CARw(t) + [CAQB^{-1} + CP\Sigma_B B^{-1} - F - A]d + [(CP\Sigma_B + CAQ)B^{-1}C - CF]\xi(t) \quad \text{(A4)}$$

The appropriate form of the identity is (Eq. 6.6 in Chien, 1985, with $E = I$ and $K = z^{-1}$):

$$CP = A + z^{-1}F \tag{A5}$$

Substituting the steady state version of Eq. A5 (since d is assumed to be constant) and premultiplying the whole equation by C^{-1}, we obtain

$$[(P\Sigma_B + AQ)B^{-1}A]y(t) = ARw(t) + A(1)Q(1)B^{-1}(1)d + [(P\Sigma_B + AQ)B^{-1}C - F]\xi(t) \tag{A6}$$

Rearrangement of Eq. A6 leads directly to Eq. 26.

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